Titles and Abstracts

Mike Hochman

Title: What does a typical Cantor system look like?

Abstract: I'll discuss what dynamical behavior is typical (in the sense of Baire category) for homeomorphisms of the Cantor set. With a few exceptions, the answers are surprisingly different from the analogous question about automorphisms of a Lebesgue space.

Christian Skau

Title: "THE ABSORPTION THEOREM FOR AFFABLE (AF-ABLE) EQUIVALENCE RELATIONS".

Abstract: Let R be a minimal AF-equivalence relation on the Cantor set X, and let Y be a closed "thin" subset of X, i.e. Y is a null set with respect to all R-invariant probability measures. We assume that the restriction of R to Y is an AF-equivalence relation S,say,on Y.We modify R in the sense that we extend S,and let R' be the resulting extended equivalence relation on X. The absorption theorem says that R' is orbit equivalent to R,and so,in particular, R' is affable.
Even in the simplest case--when Y is a finite set--this result is highly non-trivial. The absorption theorem is a powerful and crucial tool in showing that minimal Z-n(n>1) actions on the Cantor set are orbit equivalent to (minimal) Z actions.

Andres del Junco

Title: Almost continuous orbit equivalence for groups of homeomorphisms.

Abstract: Generalizing work of Sahin and myself, my student Vladimir Zhuravlev has established the following result.

Theorem: Suppose a countable amenable group G acts freely and ergodically by measure-preserving homeomorphisms of a Polish probability space X. Then the action is almost continuously orbit equivalent to the the binary odometer T acting on Y = {0,1}^N in the following sense: there are invariant co-null dense G_\delta subsets X' and Y' and measure-preserving homeomorphism \phi: X' to Y' which carries orbits onto orbits. Moreover the "co-cycles" associated with this orbit equivalence are continuous.

Alexandre Danilenko

Title: Almost continuous orbit equivalence for non-singular homeomorphisms

Abstract: This is joint work with Andres del Junco. Let X and Y be Polish spaces with non-atomic Borel measures μ and ν of full support. Suppose that T and S are ergodic non-singular homeomorphisms of (X,μ) and (Y,ν) with continuous Radon-Nikodym derivatives. If they are both either of type III_1 or of type III_λ, $0<\lambda<1$, and their `topological asymptotic ranges' are $\log\lambda\cdot\Bbb Z$ then there exists a non-singular orbit equivalence ϕ of T and S which is a homeomorphism of some invariant dense G_δ-subsets $X'\subset X$ and $Y'\subset Y$ of full measure and the Radon-Nikodym derivative $d\nu\circ\phi/d\mu$ is continuous on X'.

Andrew Dykstra

Title: Nearly continuous Kakutani equivalence

Abstract: In 2007, Del Junco, Rudolph and Weiss introduced the field of measured topological dynamics. They were interested in measurable orbit equivalences (MOE's) which are continuous on invariant subsets of full measure. Such a MOE is called nearly continuous. Somewhat surprisingly, they showed that the nearly continuous MOE relation is no more restrictive than the MOE relation.

Also they considered MOE's which restrict to topological conjugacies between induced systems. If the subset on which we induce has positive measure, then such a MOE is called an even Kakutani equivalence (EKE). If an EKE is continuous on an invariant subset of full measure it is called a weakly nearly continuous Kakutani equivalence (WNCKE). Again somewhat surprisingly, they showed that the WNCKE relation is no more restrictive than the EKE relation.

But the WNCKE relation does not require the subsets on which we induce to have any topological properties--positive measure is the only requirement. This motivates the definition of a stronger relation, called nearly continuous Kakutani equivalence (NCKE). NCKE is the same as WNCKE, except now the subsets on which we induce are required to be both of positive measure and almost clopen. This opens the door to many new classification questions. In this talk we will discuss recent progress, involving irrational rotations and odometers in particular.

Nic Ormes

Title: Polynomial Odometers

Abstract: This talk describes joint work with Sarah Bailey Frick. Polynomial odometers are adic maps on ordered Bratteli diagrams which have a regularity described by polynomials with natural coefficients. The Pascal map is the most famous example; it is is the polynomial odometer corresponding to the single polynomial x+1. These maps are continuous except on a countable set of points and with some mild additional assumptions admit uncountably many ergodic measures. We describe the ordered groups which arise for these systems, which in turn gives a description of the space of invariant measures. We can see that the unital ordered groups in this case are invariants for a kind of orbit equivalence which is continuous except on a set of universal measure 0, so are "almost continuous" with respect to all invariant measures. We show that in many cases, these unital ordered groups are isomorphic if and only if the polynomial odometers are flip conjugate by a continuous map.

Ian Putnam

Title: A new proof of the orbit equivalence classification for minimal AF-relations

Abstract: A Bratteli diagram is an infinite (locally finite) graph ; the vertices appear as an infinite sequence of vertex sets with the edges joining one to the next. The infinite path space is a totally disconnected compact space and there is a natural relation of tail equivalence: two paths are equivalent if they only differ in a finite number of entries. Such relations are called AF-relations (for approximately finite). In the case of minimal equivalence relations, these are intimately linked with Z-actions. We outline a new proof of the classification of such relations up to orbit equivalence. The original proof, given by Giordano, Skau and myself, came as a consequence of the classification of Z-actions. The key ingredient in the new proof is the same "absorption theorem" used in extending the orbit equivalence results to minimal Z, Z^2 and Z^d actions on the Cantor set.

Mrinal Roychowdhury

title: Quantization dimension functions and ergodic Markov measures

Abstract: Joint work with Dan Mauldin. For over fifty years electrical engineers and mathematicians have been interested in the problem of efficiently ``quantizing" a probability distribution in the sense of estimating a given probability by a discrete probability supported on a finite set. This problem arises in signal processing, data compression, cluster analysis, and pattern recognition, and it also has been studied in the context of economics, statistics, and numerical integration. Many useful theorems and algorithms have evolved over the years. Two main goals have been (1) finding the exact configuration of a so-called ``$n$-optimal set" which corresponds to the support of the quantized version of the distribution, and (2) estimating the rate (called ``quantization dimension") at which some specified measure of the error (also called the distortion or noise, between the quantized distribution and the

original distribution) goes to zero as n goes to infinity.
Given a Borel probability measure μ on $\D R^d$, a number $r \in (0, +\infty)$ and a natural number $n \in \D N$, the n-th \tit{quantization error} of order r for μ, is defined by
\[e_{n, r}=\te{inf}\set{\big(\int d(x, \ga)^r d\mu(x)\big)^{\frac{1}{r}} : \ga \sci \D R^d, \, \te{card}(\ga) \leq n},\]
where $d(x, \ga)$ denotes the distance from the point x to the set $\ga$ with respect to a given norm $||\cdot||$ on $\D R^d$. If $\int || x||^r d\mu(x)<\infty$ then there is some set $\ga$ for which the infimum is achieved. This set $\ga$ can then be used to give a best approximation of μ by a discrete probability supported on a set with no more than n points. The quantization dimension of order r for μ is defined to be
\[D_r=D_r(\mu) =\lim_{n \to \infty} \frac{\log n}{-\log e_{n, r}},\]
if the limit exists. If the limit does not exist then we define $\ol D_r$ as the lim sup of the sequence and $\ul D_r$ as the lim inf. In this paper quantization dimension function for the image measure of ergodic Markov measure on self similar fractals has been constructed, and a relation between the quantization dimension function and the thermodynamic formalism has been established.

Ayse Sahin

Title: Dye's Theoem in the almost continuous category.

Abstract: We prove an almost continuous version of Dye's Theorem for measure preserving homeomorphisms of separable, complete metric spaces. Hamachi and Keane established that the binary and ternary odometers are almost continuously orbit equivalent. Other special cases were established by Hamachi, Keane, Roychowdhury and Rudolph. This extension is joint work with A. del Junco.

Aimee Johnson

Title: Subactions of Z^2 Conservative Actions

Abstract: In joint work with Ayse Sahin, we consider 2-dimensional conservative actions. For such T and vectors v in Z^2, T_v may or may not be a (1-dim) conservative action. Denote by W_T the set of vectors v which yield non-conservative subactions of T. We use cutting and stacking methods to construct 2-dimensional, infinite measure preserving actions which show that certain sets W_T can be realized.